

Transfer of Quantum Correlations from One-Photon Systems to Entangled Two-Photon Systems

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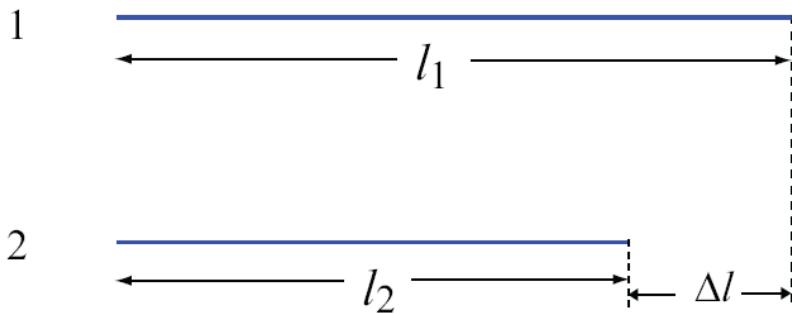
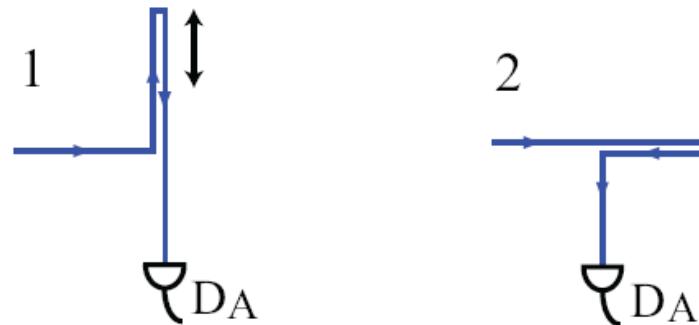
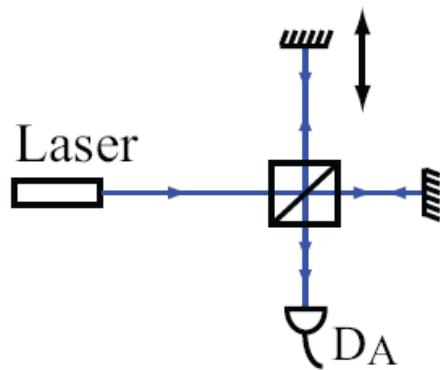
Photonics-2016, IIT Kanpur

December 6th, 2016

Outline

- Correlations?
- One-Photon System
- Entangled Two-photon System
- Transfer of Correlation
- Why is it important?

Temporal Coherence (One-Photon)



$$\Delta l = l_1 - l_2 = c \Delta\tau$$

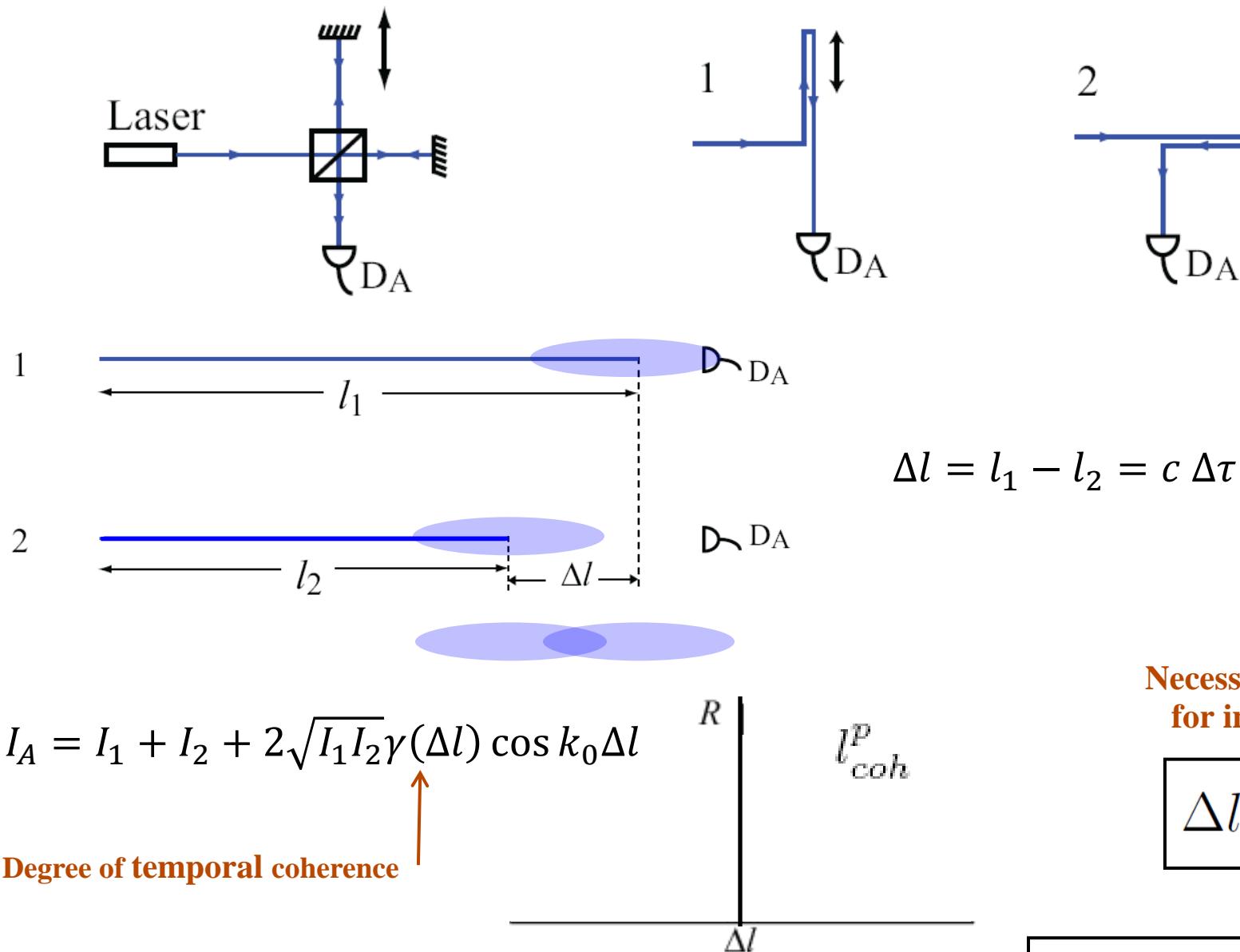
$$I_A = I_1 + I_2 + 2\sqrt{I_1 I_2} \gamma(\Delta l) \cos k_0 \Delta l$$

Degree of temporal coherence

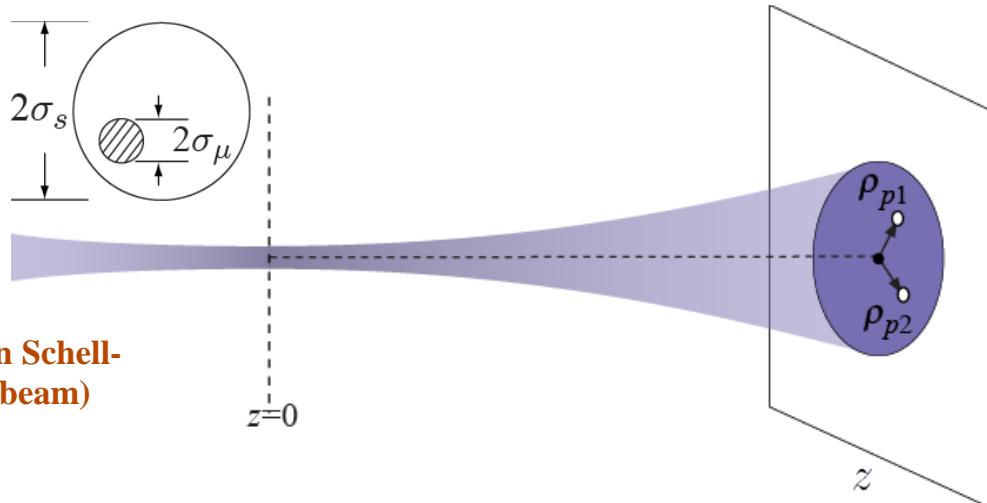
Necessary condition
for interference:

$$\Delta l < l_{coh}$$

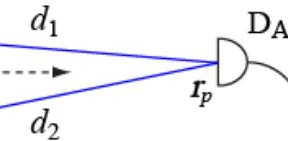
Temporal Coherence (One-Photon)



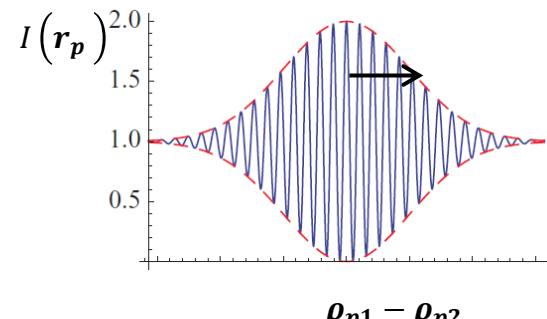
Spatial Coherence (One-Photon)



Mandel and Wolf,
Optical Coherence and Quantum Optics



Transverse coherence length



Intensity at the detector:

$$I_A = S(\rho_{p1}, z) + S(\rho_{p2}, z) + 2\sqrt{S(\rho_{p1}, z)S(\rho_{p2}, z)}\mu(\Delta\rho_p, z)\cos[(d_1 - d_2)]$$

$\Delta\rho_p = \rho_{p1} - \rho_{p2}$

Spectral density:

$$S(\rho_{p1}, z) = C \exp \left[-\frac{1}{2} \left(\frac{\rho_{p1}}{\sigma_s(z)} \right)^2 \right]$$

$$\sigma_s(z) = z \sqrt{\sigma_\mu^2 + 4\sigma_s^2} / 2k_0\sigma_s\sigma_\mu$$

Degree of spatial coherence:

$$\mu(\Delta\rho_p, z) = \exp \left[-\frac{1}{2} \left(\frac{\Delta\rho_p}{\sigma_\mu(z)} \right)^2 \right]$$

$$\sigma_\mu(z) = z \sqrt{\sigma_\mu^2 + 4\sigma_s^2} / 2k_0\sigma_s^2$$

Quantifying One-Photon Correlations

	How to quantify the correlation with two alternatives?	How to quantify the correlation of the entire field (system)?
Temporal	Degree of temporal coherence	??
Spatial	Degree of spatial coherence	??
Angular	Degree of angular coherence	??
Polarization	-	-

Polarization Correlations (one-photon)

Polarization is a two-dimensional basis.

A particular realization of the state of the field in the polarization basis

Mandel and Wolf,
Optical Coherence and Quantum Optics

$$\mathbf{E}(t) = E_H(t) \hat{H} + E_V(t) \hat{V}$$

A general mixed state of the field is:

$$\mathbf{J} = \begin{pmatrix} J_{xx} & J_{xy} \\ J_{yx} & J_{yy} \end{pmatrix} = \begin{pmatrix} \langle E_H^*(t) E_H(t) \rangle & \langle E_H^*(t) E_V(t) \rangle \\ \langle E_V^*(t) E_H(t) \rangle & \langle E_V^*(t) E_V(t) \rangle \end{pmatrix} \quad \text{Polarization matrix}$$

$$\mathbf{J} = \mathbf{J}^{\text{pol}} + \mathbf{J}^{\text{unpol}} = P \mathbf{J}^{\text{pol, unit}} + (1 - P) \mathbf{J}^{\text{unpol, unit}}$$

$$P = \frac{\text{tr } \mathbf{J}^{\text{pol}}}{\text{tr } \mathbf{J}^{\text{unpol}}} = \left[\mathbf{1} - \frac{4 \det \mathbf{J}}{(\text{tr } \mathbf{J})^2} \right]^{1/2} = V$$

$P = 0$ is the unpolarized field

$P = 1$ is the completely polarized field

P doesn't change under unitary transformation

- This is called the degree of polarization of the entire field
- Degree of polarization is *equal* to the visibility of interference fringes

Quantifying One-Photon Correlations

	How to quantify the correlation with two alternatives?	How to quantify the correlation of the entire field (system)?
Temporal	Degree of temporal coherence	??
Spatial	Degree of spatial coherence	??
Angular	Degree of angular coherence	??
Polarization	Degree of polarization	Degree of polarization

- **The correlations of a system can be quantified only if the system lives in a two-dimensional state-space**
- **Quantifying correlations in a high-dimensional state-space is an active area of research**

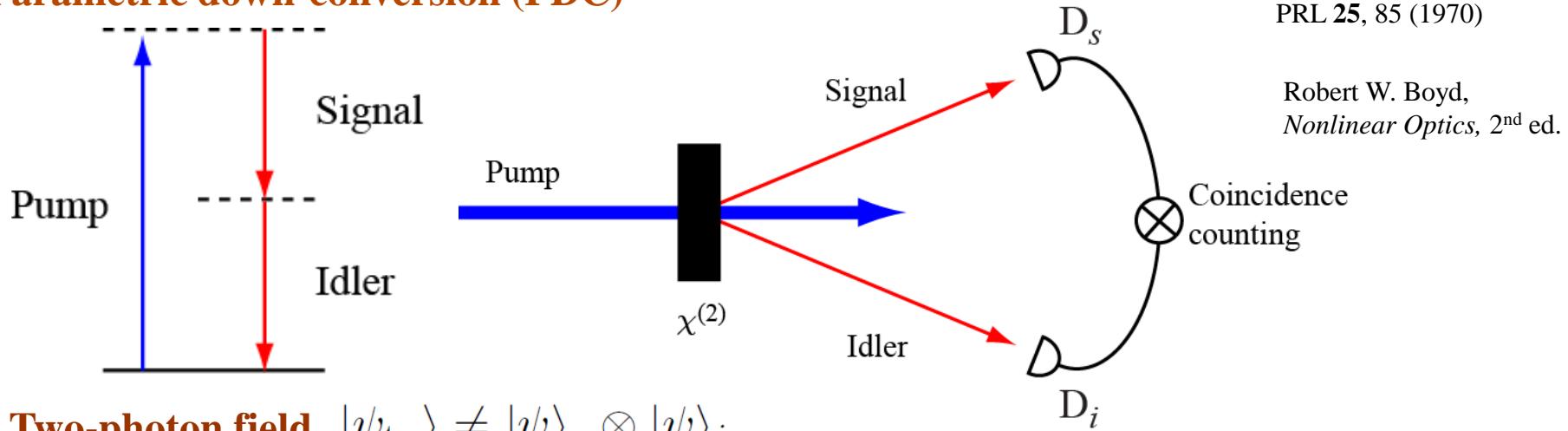
T. Baumgratz *et al.* PRL 113, 140401 (2014);
D. Girolami PRL 113, 170401 (2014)
A. Streltsov *et al.* PRL 115, 020403 (2015)

Why Study Correlations?

- Stellar interferometry (determining the size of a star, etc.)
- LIGO (Gravitational wave detection)
- Imaging
- Holography
- Lithography and Metrology
-
-
-
- Quantum cryptography and Quantum dense coding
- Quantum teleportation
- Quantum metrology (Supersensitive detection)
- Quantum Lithography
-
-
-

Entangled Two-Photon Fields

Parametric down-conversion (PDC)



Two-photon field $|\psi_{\text{tp}}\rangle \neq |\psi\rangle_s \otimes |\psi\rangle_i$

Entanglement in position and momentum

Entanglement in time and energy

Entanglement in angular position and orbital angular momentum

Entanglement in Polarization

Two-dimensional entanglement

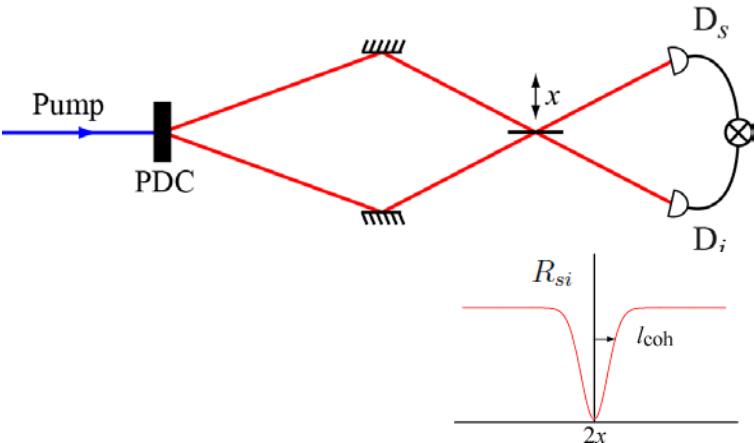
Continuous-variable
entanglement

Transfer of correlations from the pump photon to
the two entangled photons ??

Temporal Coherence (Two Entangled Photons)

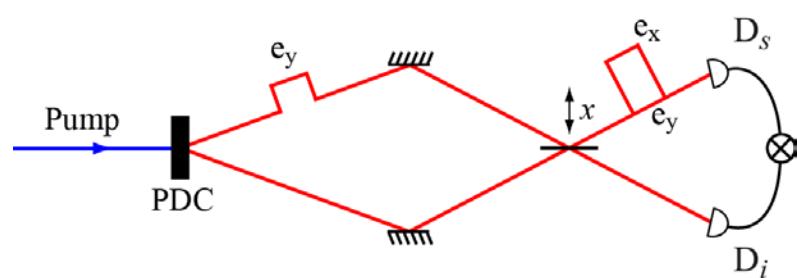
- Hong-Ou-Mandel effect

C. K. Hong et al., PRL 59, 2044 (1987)



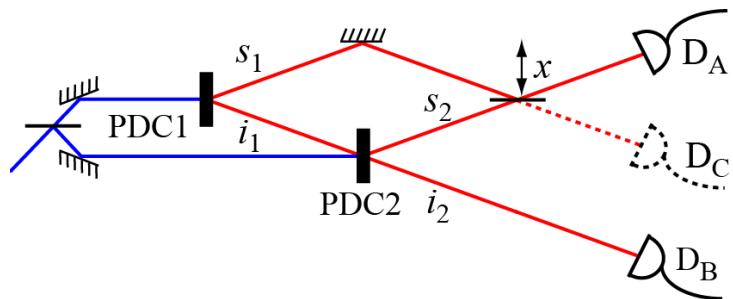
- Postponed Compensation Experiment

T. B. Pittman, PRL 77, 1917 (1996)



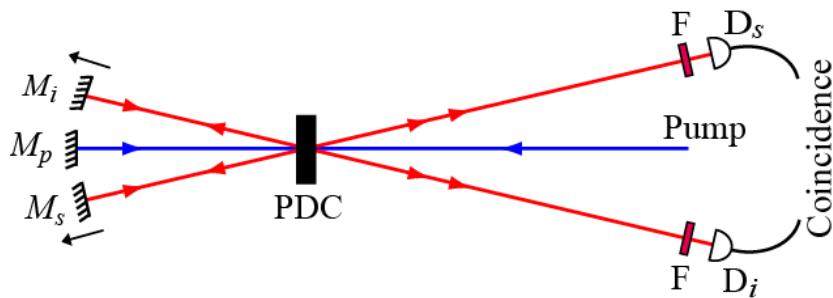
- Induced Coherence

X. Y. Zou et al., PRL 67, 318 (1991)

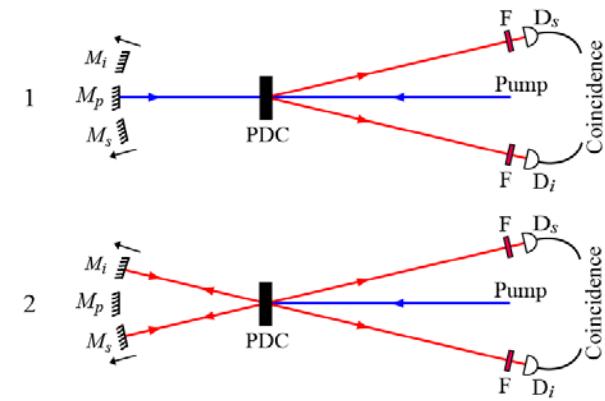
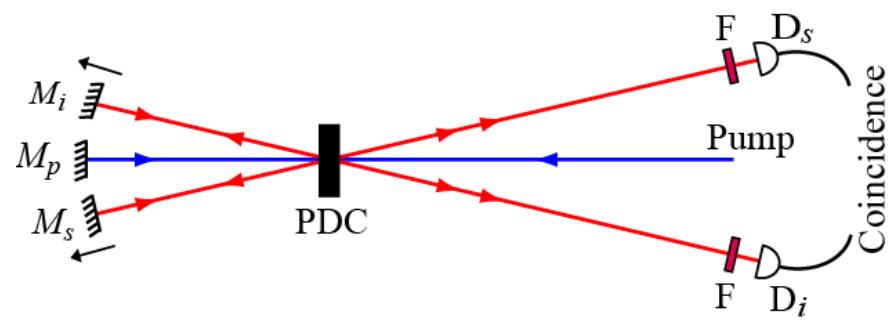


- Frustrated two-photon Creation

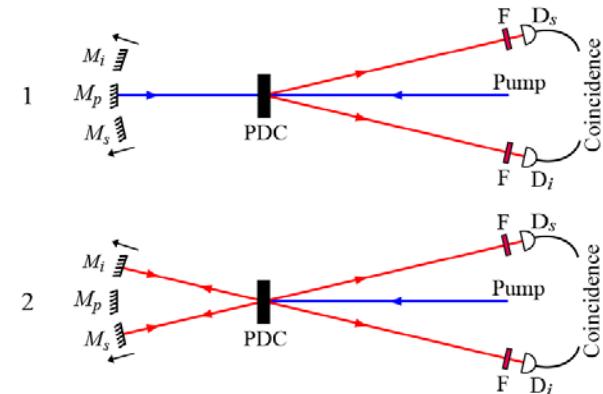
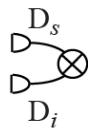
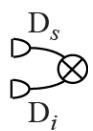
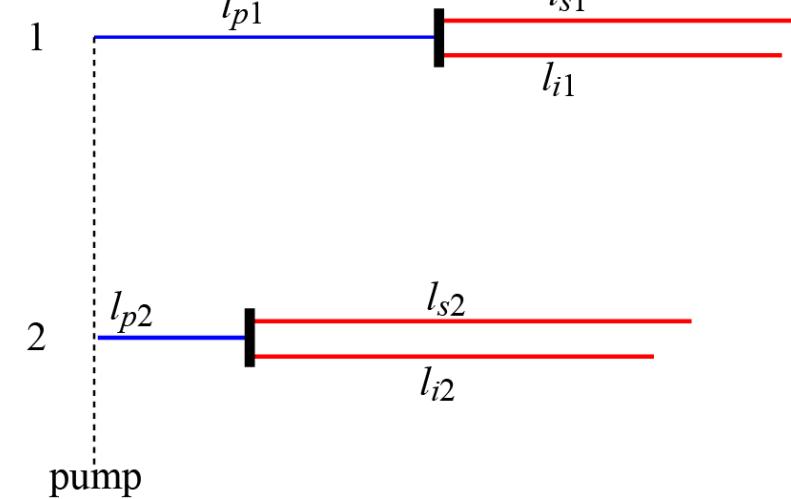
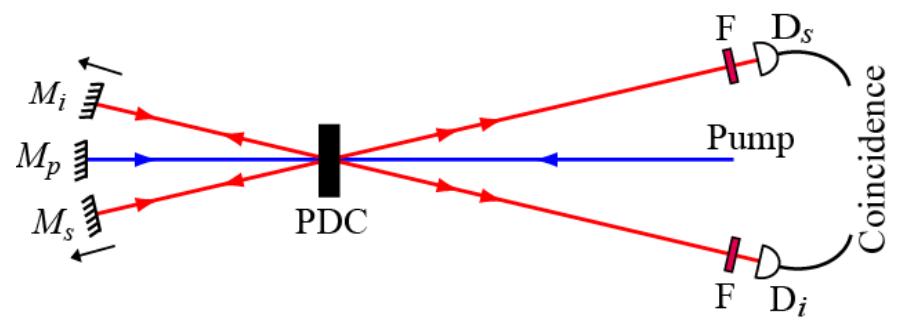
T. J. Herzog et al., PRL 72, 629 (1994)



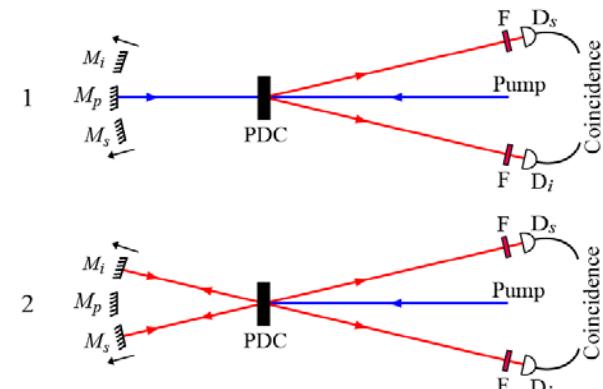
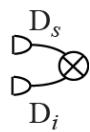
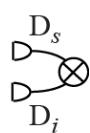
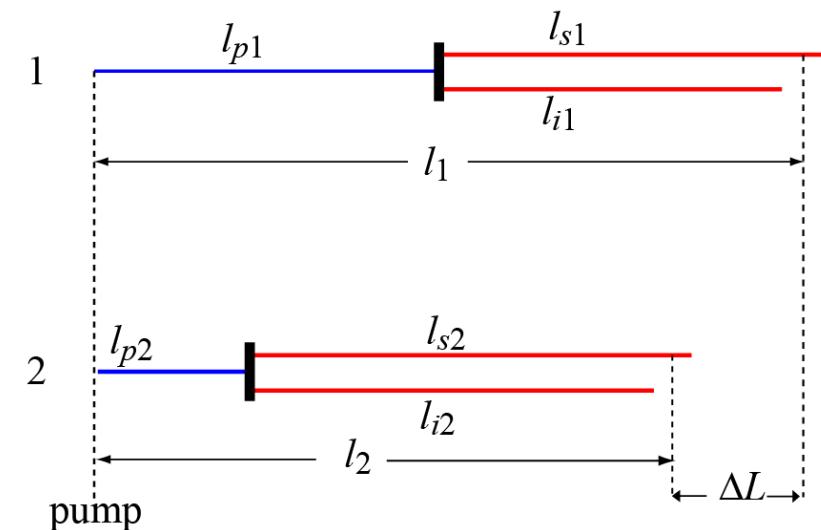
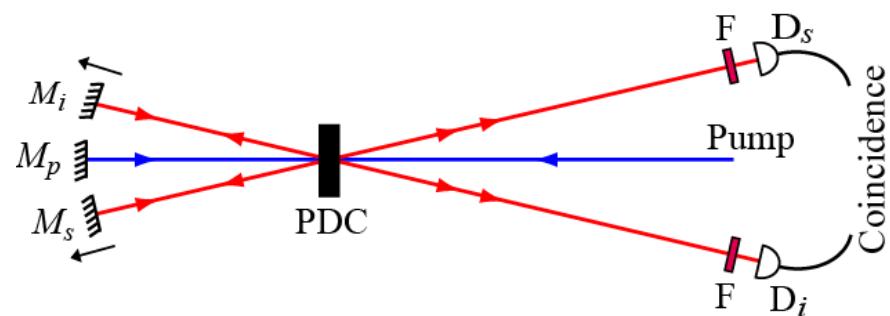
Temporal Coherence (Two Entangled Photons)



Temporal Coherence (Two Entangled Photons)

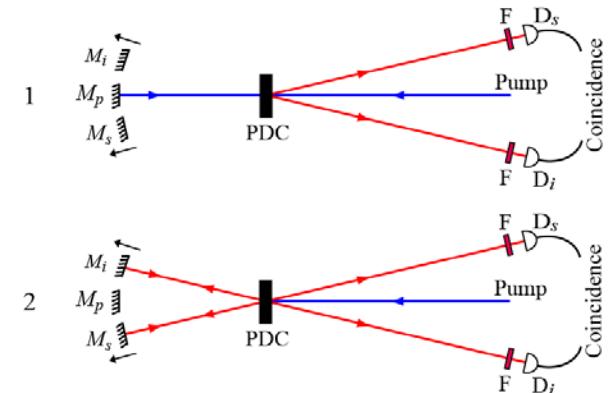
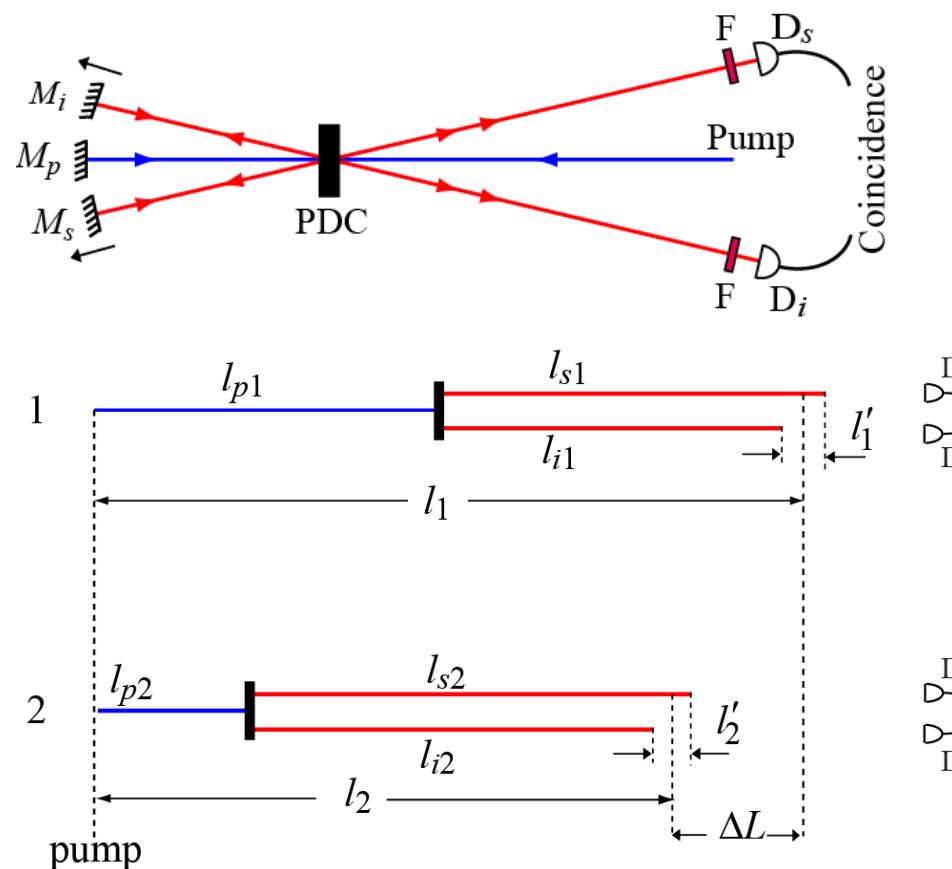


Temporal Coherence (Two Entangled Photons)



$$\Delta L = \left(\frac{l_{s1} + l_{i1}}{2} + l_{p1} \right) + \left(\frac{l_{s2} + l_{i2}}{2} + l_{p2} \right)$$

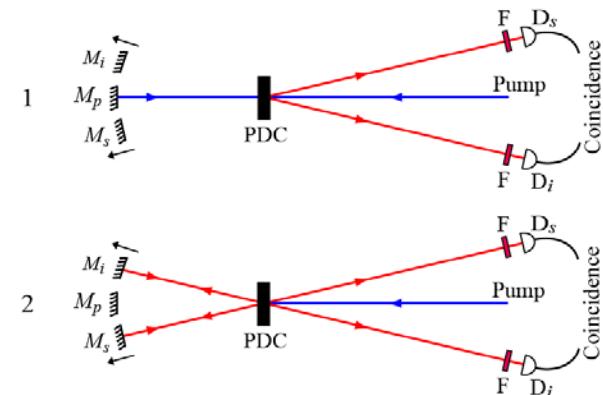
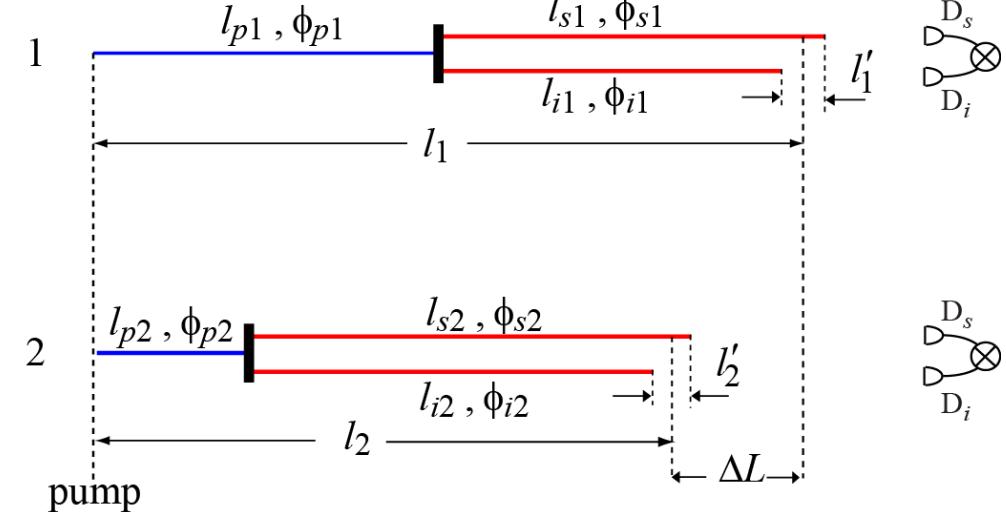
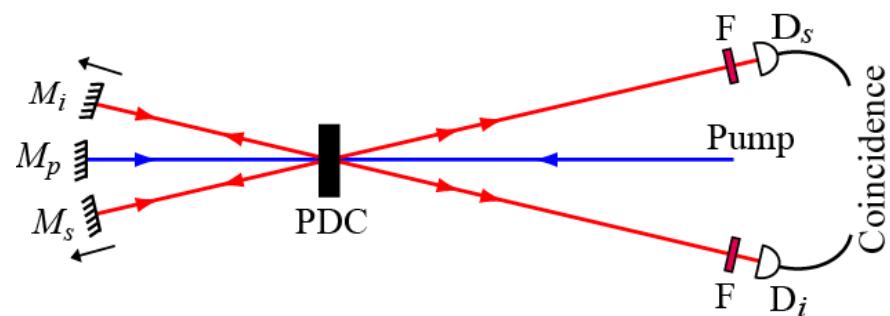
Temporal Coherence (Two Entangled Photons)



$$\Delta L = \left(\frac{l_{s1} + l_{i1}}{2} + l_{p1} \right) + \left(\frac{l_{s2} + l_{i2}}{2} + l_{p2} \right)$$

$$\Delta L' = (l_{s1} - l_{i1}) - (l_{s2} - l_{i2})$$

Temporal Coherence (Two Entangled Photons)

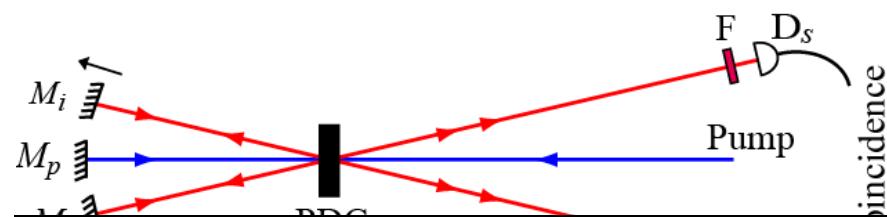


$$\Delta L = \left(\frac{l_{s1} + l_{i1}}{2} + l_{p1} \right) + \left(\frac{l_{s2} + l_{i2}}{2} + l_{p2} \right)$$

$$\Delta L' = (l_{s1} - l_{i1}) - (l_{s2} - l_{i2})$$

$$\Delta\phi = (\phi_{s1} + \phi_{i1} + \phi_{p1}) - (\phi_{s2} + \phi_{i2} + \phi_{p2})$$

Temporal Coherence (Two Entangled Photons)



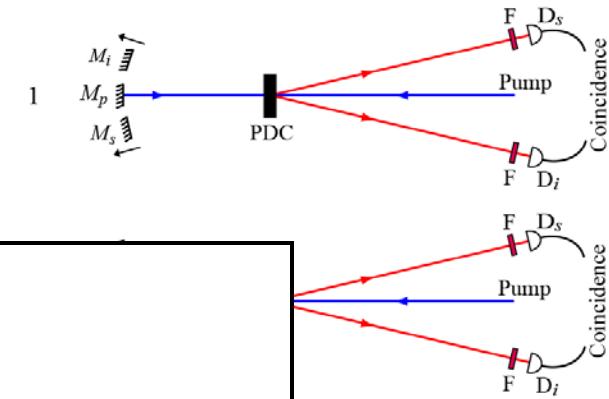
R. J. Glauber, Phys. Rev. **130**, 2529 (1963)

$$R_{si} = \left\langle \langle \psi | \hat{E}_s^{(-)}(t) \hat{E}_i^{(-)}(t + \tau) \hat{E}_i^{(+)}(t + \tau) \hat{E}_s^{(+)}(t) | \psi \rangle \right\rangle_{t, \tau}$$

$$|\psi\rangle = A \iint_0^\infty d\omega_p d\omega_s V_1(\omega_p) \Phi_1(\omega_s, \omega_p - \omega_s) e^{i(\omega_p \tau_{p1} + \phi_{p1})} |\omega_s\rangle_{s1} |\omega_p - \omega_s\rangle_{i1} \\ + A \iint_0^\infty d\omega_p d\omega_s V_2(\omega_p) \Phi_2(\omega_s, \omega_p - \omega_s) e^{i(\omega_p \tau_{p2} + \phi_{p2})} |\omega_s\rangle_{s2} |\omega_p - \omega_s\rangle_{i2}$$

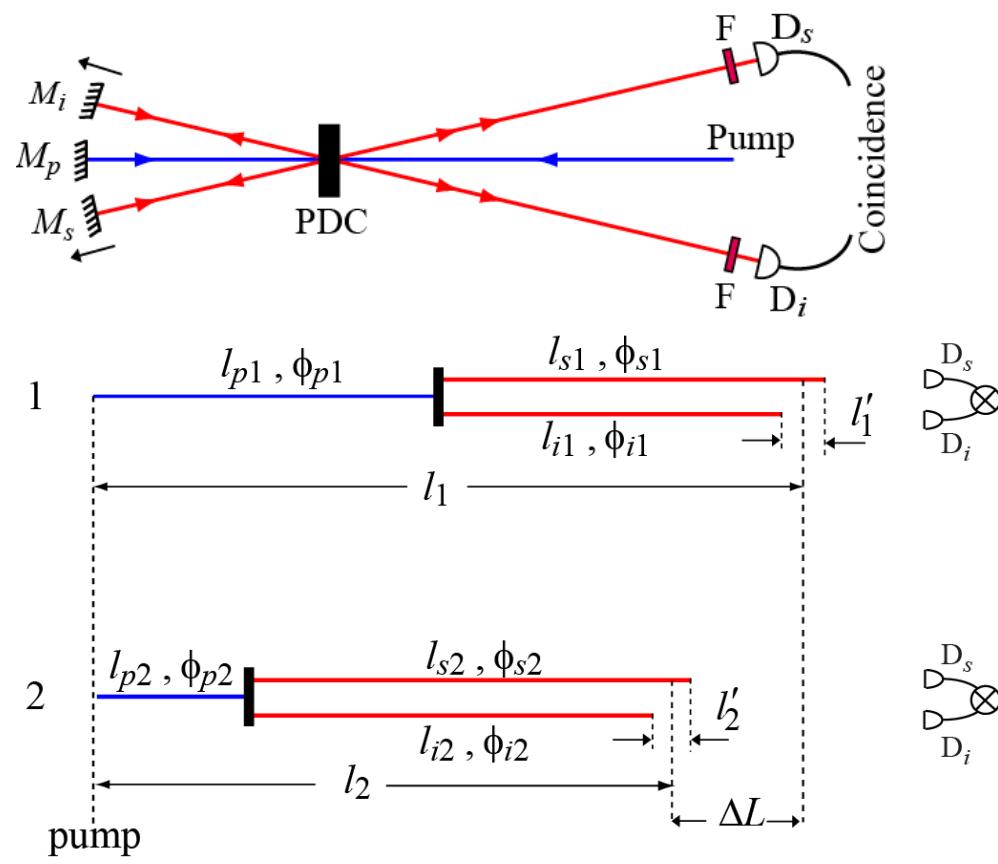
$$\hat{E}_s^{(+)}(t) = \int_0^\infty d\omega f_s(\omega - \omega_{s0}) \\ \times [c_{s1} \hat{a}_{s1}(\omega) e^{-i[\omega(t - \tau_{s1}) - \phi_{s1}]} + c_{s2} \hat{a}_{s2}(\omega) e^{-i[\omega(t - \tau_{s2}) - \phi_{s2}]}]$$

$$\hat{E}_i^{(+)}(t) = \int_0^\infty d\omega' f_i(\omega' - \omega_{i0}) \\ \times [c_{i1} \hat{a}_{i1}(\omega') e^{-i[\omega'(t - \tau_{i1}) - \phi_{i1}]} + c_{i2} \hat{a}_{i2}(\omega') e^{-i[\omega'(t - \tau_{i2}) - \phi_{i2}]}]$$



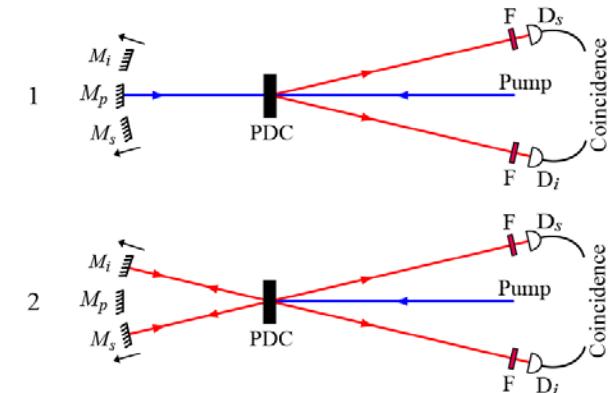
$$1 \Big) + \left(\frac{l_{s2} + l_{i2}}{2} + l_{p2} \right) \\ l_{s2} - l_{i2}) \\ p_1) - (\phi_{s2} + \phi_{i2} + \phi_{p2})$$

Temporal Coherence (Two Entangled Photons)



$$R_{si} = R_1 + R_2 + 2\sqrt{R_1 R_2} \gamma'(\Delta L') \gamma(\Delta L) \cos(k_0 \Delta L + \Delta \phi)$$

Degree of two-photon temporal coherence



$$\Delta L = \left(\frac{l_{s1} + l_{i1}}{2} + l_{p1} \right) + \left(\frac{l_{s2} + l_{i2}}{2} + l_{p2} \right)$$

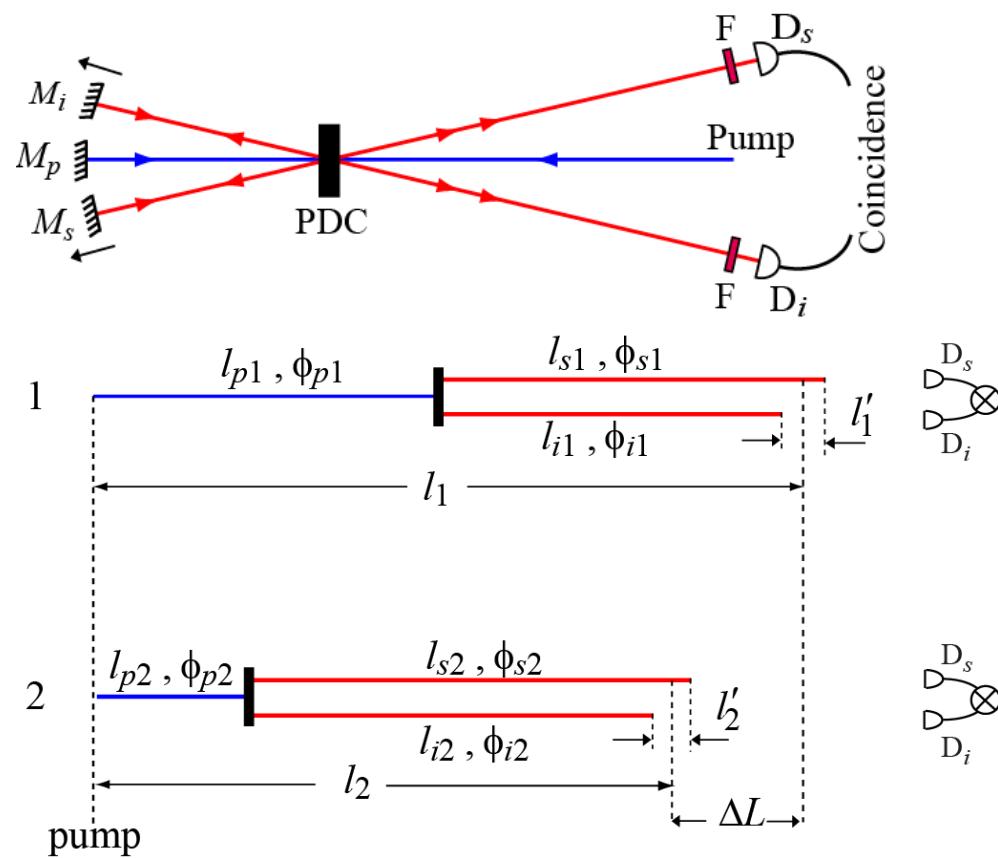
$$\Delta L' = (l_{s1} - l_{i1}) - (l_{s2} - l_{i2})$$

$$\Delta \phi = (\phi_{s1} + \phi_{i1} + \phi_{p1}) - (\phi_{s2} + \phi_{i2} + \phi_{p2})$$

Necessary conditions for two-photon interference:

$\Delta L < l_{coh}^p$
$\Delta L' < l_{coh}$

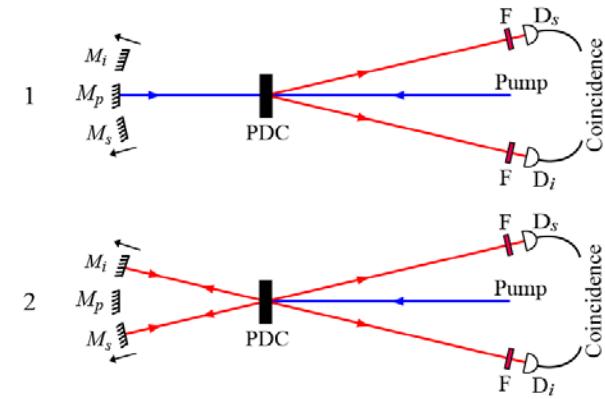
Temporal Coherence (Two Entangled Photons)



$$R_{si} = R_1 + R_2 + 2\sqrt{R_1 R_2} \gamma'(\Delta L') \gamma(\Delta L) \cos(k_0 \Delta L + \Delta \phi)$$

When : $\Delta L' = 0$

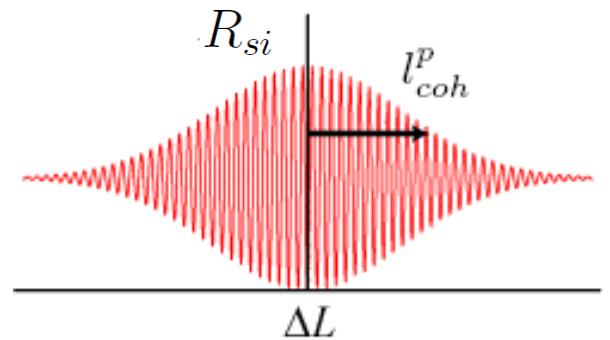
- the temporal coherence properties of the pump gets transferred to the entangled two-photon field



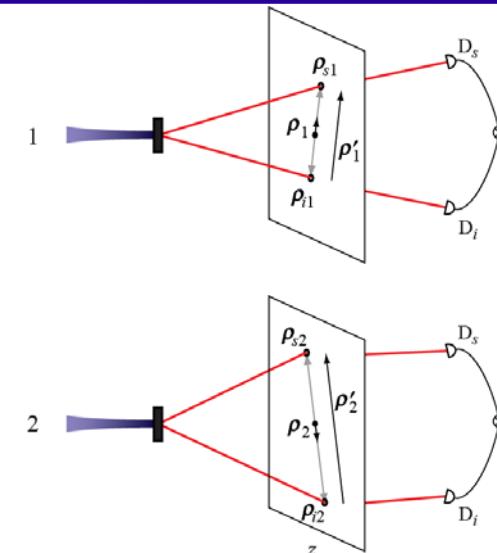
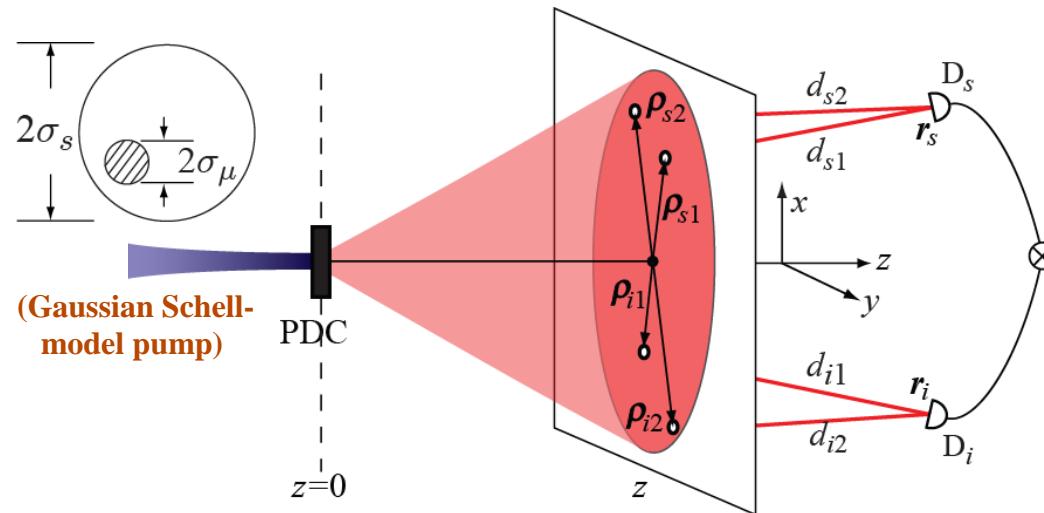
$$\Delta L = \left(\frac{l_{s1} + l_{i1}}{2} + l_{p1} \right) + \left(\frac{l_{s2} + l_{i2}}{2} + l_{p2} \right)$$

$$\Delta L' = (l_{s1} - l_{i1}) - (l_{s2} - l_{i2})$$

$$\Delta \phi = (\phi_{s1} + \phi_{i1} + \phi_{p1}) - (\phi_{s2} + \phi_{i2} + \phi_{p2})$$



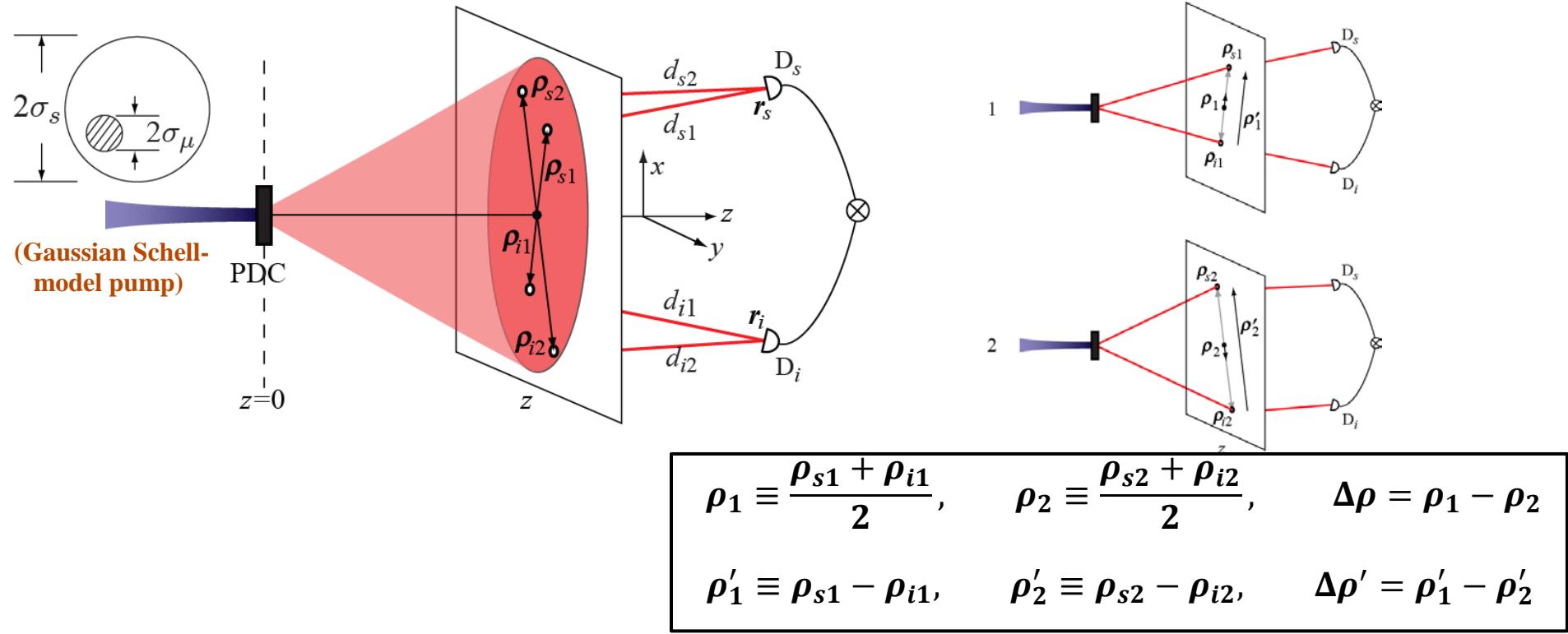
Spatial Coherence (Two Entangled photons)



$$\rho_1 \equiv \frac{\rho_{s1} + \rho_{i1}}{2}, \quad \rho_2 \equiv \frac{\rho_{s2} + \rho_{i2}}{2}, \quad \Delta\rho = \rho_1 - \rho_2$$

$$\rho'_1 \equiv \rho_{s1} - \rho_{i1}, \quad \rho'_2 \equiv \rho_{s2} - \rho_{i2}, \quad \Delta\rho' = \rho'_1 - \rho'_2$$

Spatial Coherence (Two Entangled photons)



Two-photon coincidence intensity:

$$R_{si}(\mathbf{r}_s, \mathbf{r}_i) = S^{(2)}(\rho_1, z) + S^{(2)}(\rho_2, z) + 2\sqrt{S^{(2)}(\rho_1, z)S^{(2)}(\rho_2, z)}\mu(\Delta\rho, z)\cos(k_0\Delta L + \Delta\phi)$$

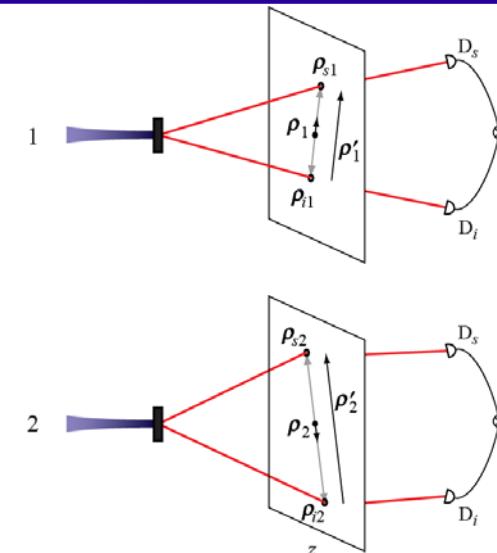
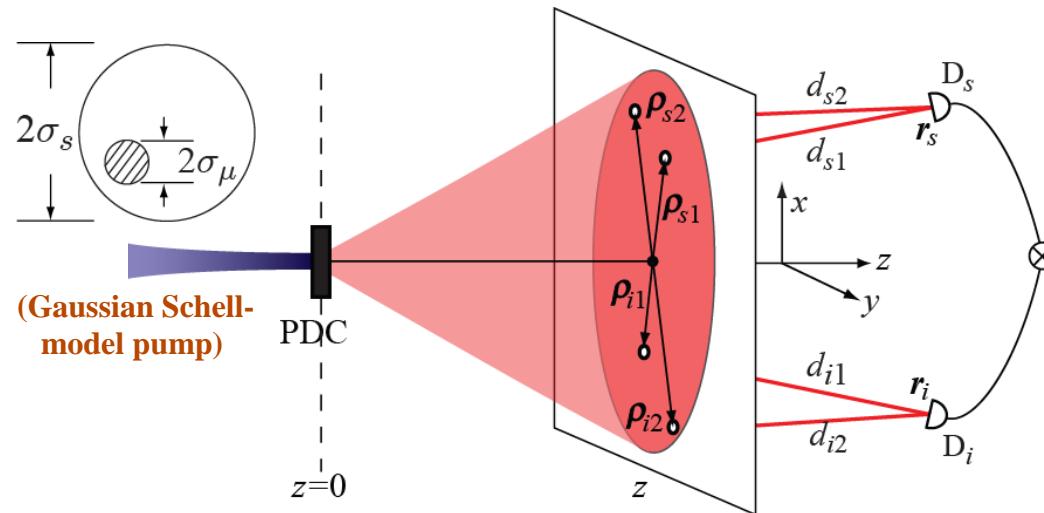
Two-photon Spectral density: $S^{(2)}(\rho_1, z) = C \exp \left[-\frac{1}{2} \left(\frac{\rho_1}{\sigma_s(z)} \right)^2 \right]$

Necessary condition for interference:

Degree of two-photon spatial coherence: $\mu^{(2)}(\Delta\rho, z) = \exp \left[-\frac{1}{2} \left(\frac{\Delta\rho}{\sigma_\mu(z)} \right)^2 \right]$

$$|\Delta\rho| < \sigma_\mu(z)$$

Spatial Coherence (Two Entangled photons)



the spatial coherence properties of the pump gets transferred to the entangled two-photon field

$$\rho_1 \equiv \frac{\rho_{s1} + \rho_{i1}}{2}, \quad \rho_2 \equiv \frac{\rho_{s2} + \rho_{i2}}{2}, \quad \Delta\rho = \rho_1 - \rho_2$$

$$\rho'_1 \equiv \rho_{s1} - \rho_{i1}, \quad \rho'_2 \equiv \rho_{s2} - \rho_{i2}, \quad \Delta\rho' = \rho'_1 - \rho'_2$$

Two-photon coincidence intensity:

$$R_{si}(\mathbf{r}_s, \mathbf{r}_i) = S^{(2)}(\rho_1, z) + S^{(2)}(\rho_2, z) + 2\sqrt{S^{(2)}(\rho_1, z)S^{(2)}(\rho_2, z)}\mu(\Delta\rho, z)\cos(k_0\Delta L + \Delta\phi)$$

Two-photon Spectral density:

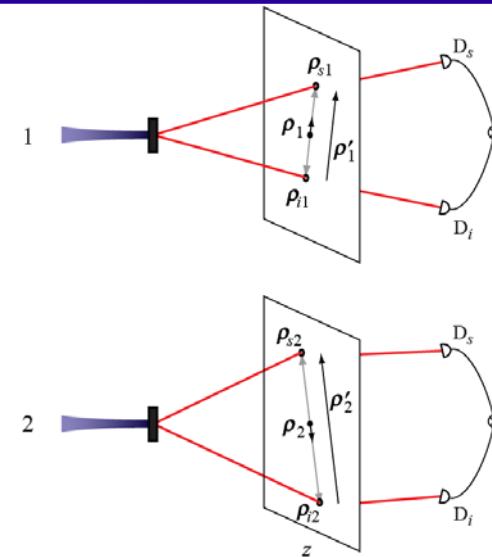
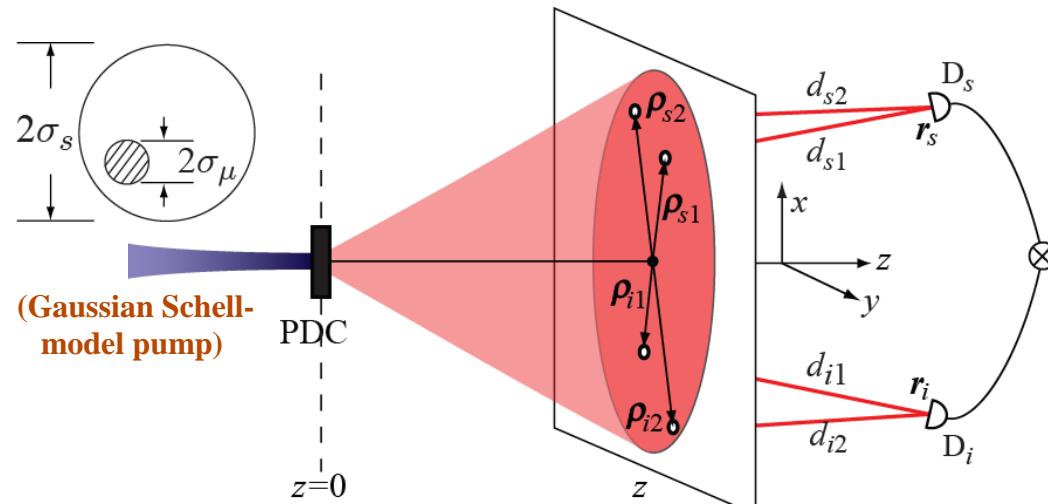
$$S^{(2)}(\rho_1, z) = C \exp \left[-\frac{1}{2} \left(\frac{\rho_1}{\sigma_s(z)} \right)^2 \right]$$

Necessary condition for interference:

$$|\Delta\rho| < \sigma_\mu(z)$$

Degree of two-photon spatial coherence: $\mu^{(2)}(\Delta\rho, z) = \exp \left[-\frac{1}{2} \left(\frac{\Delta\rho}{\sigma_\mu(z)} \right)^2 \right]$

Quantifying Entanglement of Spatial two-qubit States



Entangled two-qubit state

$$\rho_{\text{qubit}} = \begin{pmatrix} a & 0 & 0 & c \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ d & 0 & 0 & b \end{pmatrix}$$

O'Sullivan et al., PRL **94**, 220501 (2005)
 Neves et al., PRA **76**, 032314 (2007)
 Walborn et al., PRA **76**, 062305 (2007)
 Taguchi et al., PRA **78**, 012307 (2008)

Entanglement of the state (Concurrence) :

$$C(\rho_{\text{qubit}}) = 2|c| = V \leq \mu(\Delta\rho, z)$$

Concurrence W. K. Wootters, PRL **80**, 2245 (1998)

$$\zeta = \rho (\sigma_y \otimes \sigma_y) \rho^* (\sigma_y \otimes \sigma_y) \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$C(\rho) = \max \{0, \sqrt{\alpha_1} - \sqrt{\alpha_2} - \sqrt{\alpha_3} - \sqrt{\alpha_4}\}$$

A. K. Jha, G.A. Tyler and R.W. Boyd, PRA **81**, 053832 (2010)

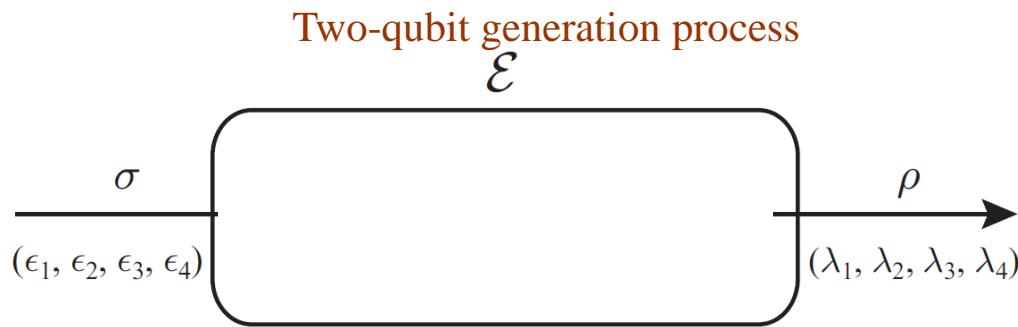
A. K. Jha et al., PRL **104**, 010501 (2010)

Transfer of correlations in an unrestricted Hilbert space ??

Transfer of correlations in an unrestricted Hilbert space ??

	How to quantify the correlation with two alternatives?	How to quantify the correlation of the entire field (system)?	How to quantify entanglement ?
Spatial	Degree of spatial two-photon coherence	??	Concurrence (two-qubit states)
Spatial	Degree of spatial two-photon coherence	??	Concurrence (two-qubit states)
Angular	Degree of angular two-photon coherence	??	Concurrence (two-qubit states)
Polarization	-	-	Concurrence

Transfer of correlations



A polarization entangled two-photon state lives in a four-dimensional space with the basis vectors

$$\{|H\rangle_s|H\rangle_i, |H\rangle_s|V\rangle_i, |V\rangle_s|H\rangle_i, |V\rangle_s|V\rangle_i\}$$

$$J = \begin{pmatrix} \langle E_H^*(t)E_H(t) \rangle & \langle E_H^*(t)E_V(t) \rangle \\ \langle E_V^*(t)E_H(t) \rangle & \langle E_V^*(t)E_V(t) \rangle \end{pmatrix}$$



$$\sigma \equiv \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \otimes J$$

Eigenvalues in decreasing order

$$(\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4) \equiv ((1+P)/2, (1-P)/2, 0, 0)$$

Degree of polarization (P)

??

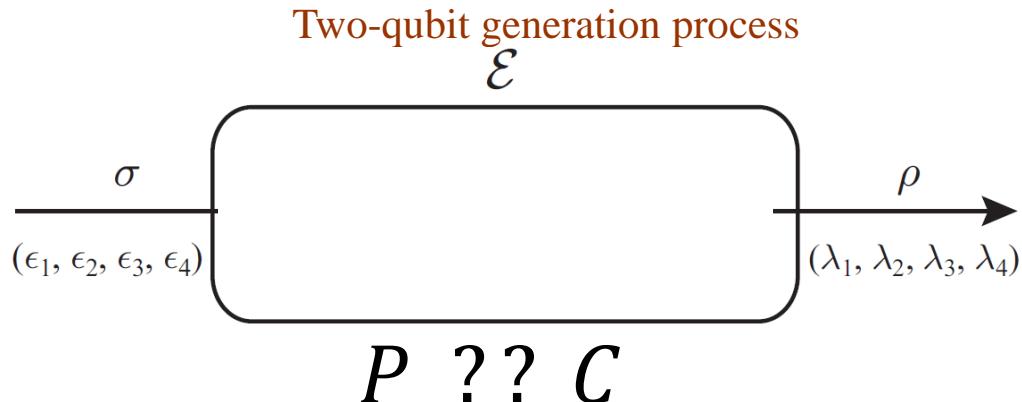
Concurrence (C)

$$\rho = \begin{pmatrix} \rho_{11} & \rho_{12} & \rho_{13} & \rho_{14} \\ \rho_{21} & \rho_{22} & \rho_{23} & \rho_{24} \\ \rho_{31} & \rho_{32} & \rho_{33} & \rho_{34} \\ \rho_{41} & \rho_{42} & \rho_{43} & \rho_{44} \end{pmatrix}$$



Eigenvalues in decreasing order
(λ₁, λ₂, λ₃, λ₄)

Transfer of correlations



If the process $\sigma \rightarrow \rho$ is: (1) trace preserving and (2) entropy non-decreasing.

Then: ρ is majorized by σ :

Quantum Computation and Quantum Information, M.
Nielsen, I. Chuang, (Cambridge University Press)(2003).

$$\begin{aligned}\lambda_1 &\leq \epsilon_1, \\ \lambda_1 + \lambda_2 &\leq \epsilon_1 + \epsilon_2, \\ \lambda_1 + \lambda_2 + \lambda_3 &\leq \epsilon_1 + \epsilon_2 + \epsilon_3, \\ \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 &= \epsilon_1 + \epsilon_2 + \epsilon_3 + \epsilon_4\end{aligned}$$

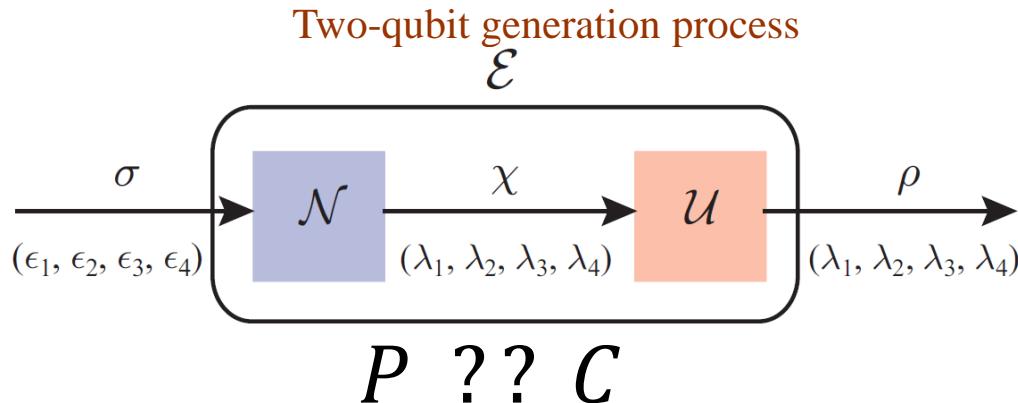
Two-qubit state ρ under global unitary transformation obeys

$$C(\rho) \leq \max\{0, \lambda_1 - \lambda_3 - 2\sqrt{\lambda_2 \lambda_4}\}$$

Phys. Rev. A **62**, 022310 (2000)

Phys. Rev. A **64**, 012316 (2001)

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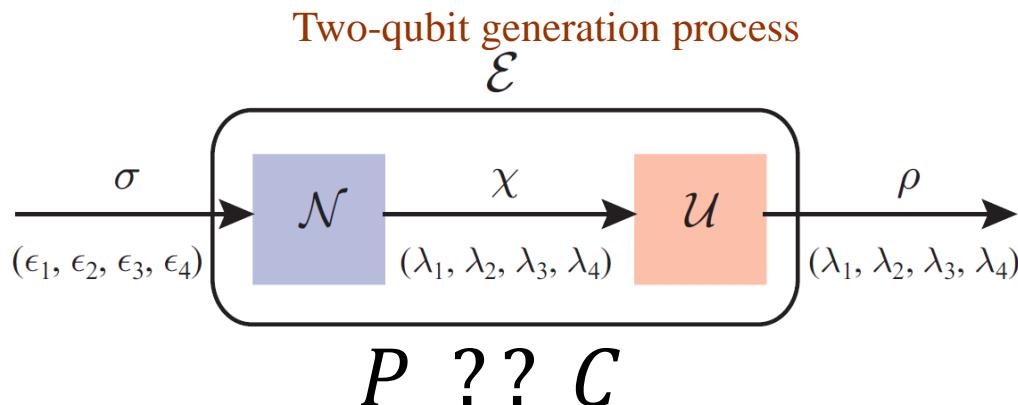
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Phys. Rev. A **62**, 022310 (2000)

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$$\lambda_1 \leq \epsilon_1 = \frac{1+P}{2}$$

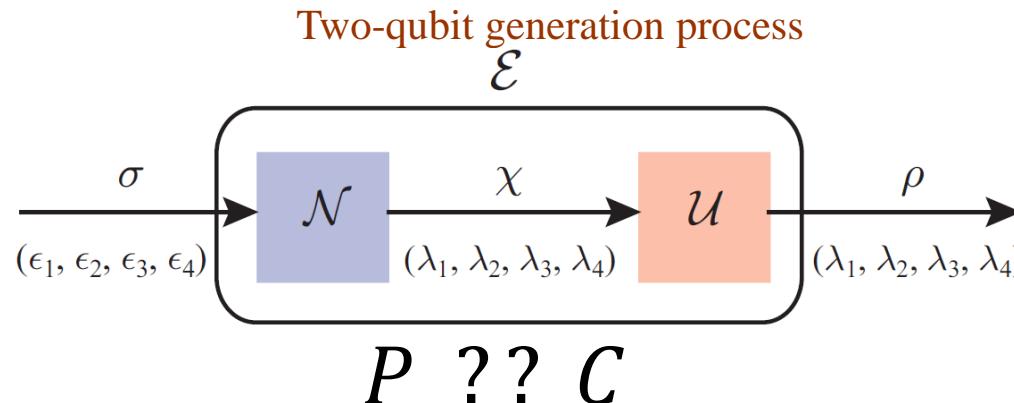
But $C(\rho) \leq \lambda_1$

Therefore,

$$C(\rho) \leq \frac{1+P}{2}$$

Polarization correlation of the one-photon pump field gets transferred and manifest as the two-photon entanglement. The maximum manifestation is bounded by the degree of polarization.

Transfer of correlations (The case of “2D” two-qubit state)



If the process $\sigma \rightarrow \rho$ is: (1) trace preserving and (2) entropy non-decreasing.

Then: ρ is majorized by σ :

$$\lambda_1 \leq \epsilon_1$$

“2D” two-qubit state ($\lambda_3 = \lambda_4 = 0$) $\rho^{(2D)} = \tilde{P}|\psi^{(2D)}\rangle\langle\psi^{(2D)}| + (1 - \tilde{P})\bar{\mathbb{1}}^{(2D)}$

Concurrence is a convex function
on the space of density matrix

$$C(\sum_i p_i \rho_i) \leq \sum_i p_i \hat{C}(\rho_i) \quad 0 \leq p_i \leq 1$$

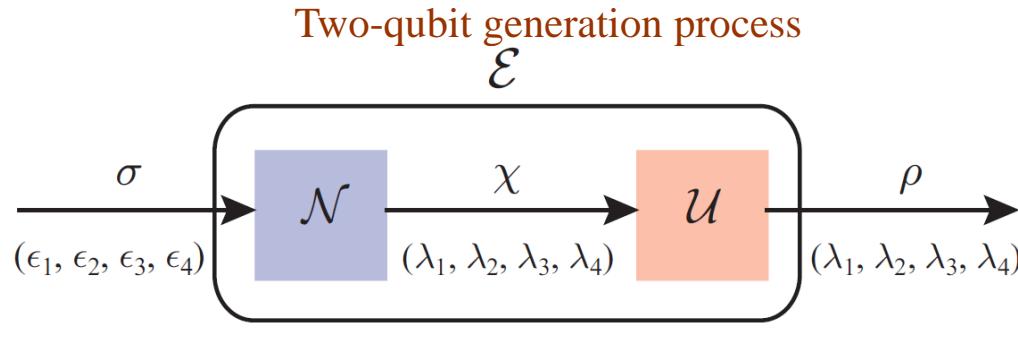
$$\sum_i p_i = 1$$

Therefore $C(\rho^{(2D)}) \leq \tilde{P}$

But $\tilde{P} = \lambda_1 - \lambda_2 = 2\lambda_1 - 1 \leq 2\epsilon_1 - 1 = \epsilon_1 - \epsilon_2 = P$

Hence $C(\rho^{(2D)}) \leq P$

Transfer of correlations (The case of “2D” two-qubit state)



P ? ? C

The general bound

$$C(\rho) \leq \frac{1+P}{2}$$

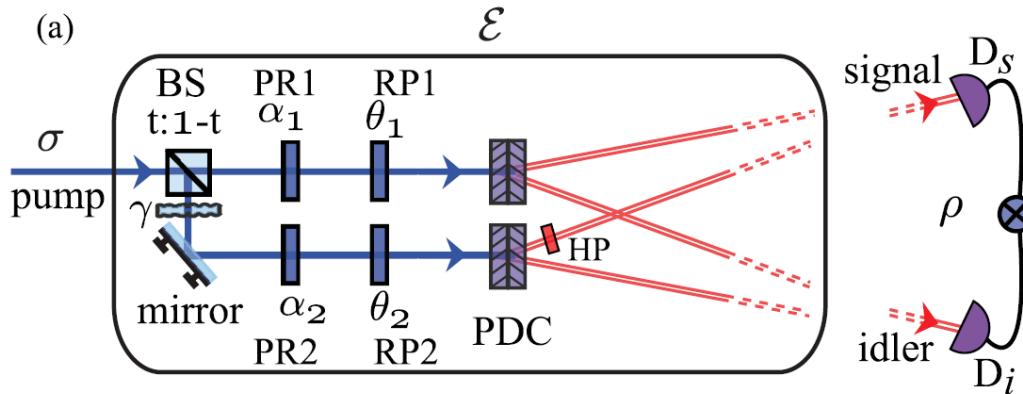
Bound for 2D states

$$C(\rho^{(2D)}) \leq P$$

G. Kulkarni, V. Subrahmanyam, and A. K. Jha,
Phys. Rev. A **93**, 063842 (2016)

Transfer of correlations (a numerical experiment)

Two-qubit generation process



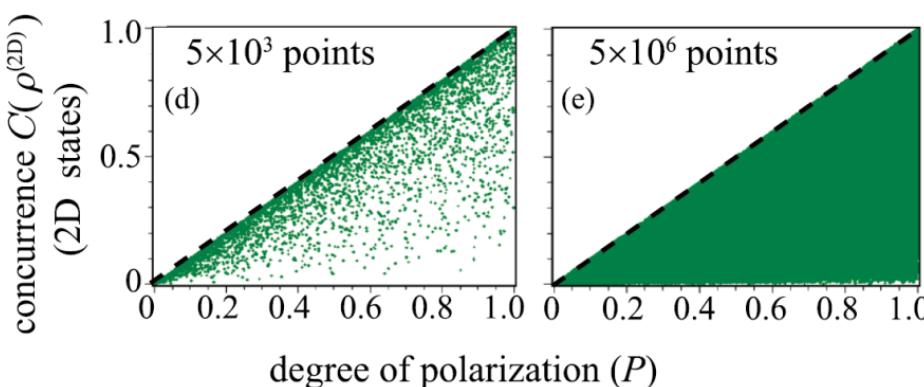
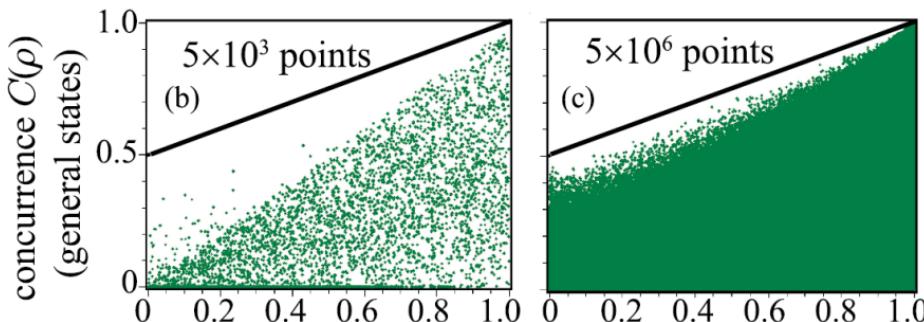
P ?? C

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$$C(\rho) \leq \frac{1+P}{2}$$

Bound for 2D states

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Conclusions

- In parametric down-conversion, the coherence properties of the pump photon gets transferred to the entangled two-photon field.
- For the polarization degree of freedom, one-photon correlations set an upper bound on two-photon polarization entanglement.

The general bound

$$C(\rho) \leq \frac{1+P}{2}$$

Bound for 2D states

$$C(\rho^{(2D)}) \leq P$$

- This needs to be extended to include high-dimensional and continuous variable entanglements.

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IIT Kanpur



Thank you for your attention